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First-Order Reliability Application and Verification Methods for Semistatic Structures

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Escalating risks of aerostructures stimulated by increasing size, complexity, and cost should no longer be ignored in conventional deterministic safety design methods. The deterministic pass-fail concept is incompatible with probability and risk assessments; stress audits are shown to be arbitrary and incomplete, and the concept compromises the performance of high-strength materials. A reliability method is proposed that combines first-order reliability principles with deterministic design variables and conventional test techniques to surmount current deterministic stress design and audit deficiencies. Accumulative and propagation design uncertainty errors are defined and appropriately implemented into the classical safety-index expression. The application is reduced to solving for a design factor that satisfies the specified reliability and compensates for uncertainty errors, and then using this design factor as, and instead of, the conventional safety factor in stress analyses. The resulting method is consistent with current analytical skills and verification practices, the culture of most designers, and the development of semistatic structural designs.

Nomenclature

e = uncertainty cumulative error
F = uniaxial stress, ksi
K = factor in resistive-stress probability and confidence range
N = factor in applied-stress probability and confidence range
n = statistical sample size
p = probability of failure, $1 - R$
R = reliability
SF = conventional safety factor
Z = reliability safety index
η = coefficient of variation, σ/μ
λ = stress distribution zone, ksi
μ = statistical mean, ksi
σ = standard deviation, ksi
ϕ = compensating coefficient

Subscripts

A = applied variable
D = "design-to" variable
e = uncertainty error
R = resistive variable
T = test-derived variable
0 = stress margin zone

Introduction

THE primary purposes of metallic and composite structures are to sustain operational environments with no detrimental deformation, reliably over a specified duration, and to achieve it all at least cost. These integrating and controlling design imperatives are the essence of robustness and structural integrity. Conventional deterministic stress audits have been shown to be arbitrary, incomplete, and often misleading for the reliability and costs of future aerostructures. There are many generic probability techniques investigated and evolving[1] for determining and providing reliable structures, but the first-order reliability method proposed here is more compatible with current system design and verification practices for semistatic structures. Semistatic structures respond to static and dynamic environments, and constitute over 60% of the structural weight of launch vehicles, spacecraft, and experiments. This type of structure includes liquid-propellant tanks, thrust mounts, solid-rocket motor cases, shrouds, main frames, and supports.

Because of the dominant influence of semistatic structure on delivery performance, and importance of supporting risk analyses and reducing design time, this study was extended over the previous investigation[2] to craft a most direct and user-friendly design and verification approach. The first-order reliability technique using design variables and verification methods in common with the conventional deterministic method was pursued because of its conformity, expedience, design simplicity, and ease of transition. The study further defined and appropriately implemented uncertainty design variables, and introduced reliability and error-compensating coefficient to provide a more versatile system reliability analysis, verification, and safety audit. The proposed first-order reliability method was independently verified with the classical first-order approach. The weight advantage of high-strength material structures is conserved to improve recurring delivery performance.

Failure Concept

Failure occurs when the applied stress on a structure exceeds the resistive stress of the structural material. This failure concept integrates the statistical interfaces of the measured uniaxial material resistive yield stress with the design applied stresses, using measured and assumed environmental data and manufacturing tolerances. Applied stresses are multiaxially induced and converted into uniaxial stresses through the minimum-distortion-energy theory so as to be compatible with the interfacing uniaxial resistive stress. The probability nature of these interfaces is defined through probabilistic density distributions illustrated in Fig. 1.

The overlap of the two distribution tails determines the probability that a weak resistive material will encounter an excessive applied stress and undergo failure. The probability of failure is reduced as the tail overlap area is reduced. The overlap area is governed by the difference of the distribution means and their shapes. The first-order reliability method decreases the tail interference by increasing the difference of the resistive- and applied-stress means, $\mu_R - \mu_A$, of their symmetrical shape function. The conventional deterministic method decreases the tail interference by increasing the difference of the resistive- and applied-stress means, and through the safety factor[3] expressed as the ratio of the minimum material strength and maximum allowed applied stress,

$$SF = \frac{F_R}{F_A} \tag{1}$$

The safety factor essentially extends the applied-stress range fac-

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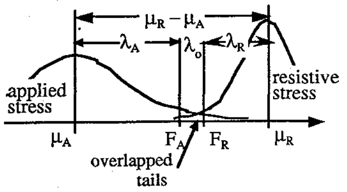

Fig. 1 Concept of failure probability.

tor. The deterministic method ignores the distribution shape in the overlap area.

The deterministic concept provides an incomplete system safety assessment because Eq. (1) does not include the material and load probability contributions in zones λ_R and λ_A, as illustrated in Fig. 1. The outgrowth of this deficiency is that a higher safety factor may provide a more reliable structure than a lower factor, but not necessarily if the lower-safety-factor structure is specified with higher material and design load probabilities. An arbitrarily and independently selected high limit-load probability would seem to provide a safer structure than one with a lower probability, but not if the combined dispersions of the applied and resistive stresses are larger than those with a lower load probability. Consequently, deterministic safety factors are incapable of comparing the total safety of one structural region with another having different applied stress distributions.

Another weakness in the deterministic method is that imposing a universal safety factor on all structural materials compromises high-strength materials. Substituting the ratio of the minimum material strength to the maximum allowed design stress of Eq. (1) into the expression for their difference, we have

$$F_R - F_A = F_R\left(1 - \frac{1}{\mathrm{SF}}\right) \tag{2}$$

The difference is seen to increase proportionately with increasing strengths of the selected materials, though retaining a constant safety factor. This increasing stress difference decreases the allowed applied stress of high-strength materials, which denies the structure more available operational elastic stress. In effect, the denied elastic strength squanders technology gains of high-strength metallurgy, promotes the selection of low-performance materials, and unnecessarily increases the weight of high-strength aerostructures, all of which increase the recurring payload performance cost.

Furthermore, the present pass-fail safety factor is arbitrary and does not support the current risk criteria, simply defined as the product of probability of failure and cost consequences. The proposed first-order reliability method surmounts these deficiencies and improvises a verification technique using current deterministic standards and experimental data.

First-Order Reliability Concept

The first-order reliability concept assumes that the applied- and resistive-stress probability density functions are normal and independent. Normal distributions are overwhelmingly observed in structural data and are justified by the central limit theorem. The normal distribution assumption allows the statistical characterization of random variables to be completely and expediently determined by the mean and standard deviation. Normal-distribution techniques are the best developed and the easiest to learn and apply. To employ other distributions for small sample sizes is to prematurely consider unnecessary and burdensome statistical information. Note in Fig. 1 that only the worst-case sides of the two distributions are involved in the failure concept. Hence, where a phenomenon is known to be non-normal, the distribution may be split, with the mode (peak frequency point) representing the mean. The standard deviation is then calculated from the relevant sides, which are assumed to be symmetrical about the peak frequency. Unusual disparity may be treated like other design uncertainties. This normalization of skewed distributions amounts to trading one design assumption for another: trading a little eloguence for reduced labor and lead time.

Accordingly, the assumed normal and independent probability density functions applied to Fig. 1 may be combined to form a third normal expression,[4]

$$Z = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} \tag{3}$$

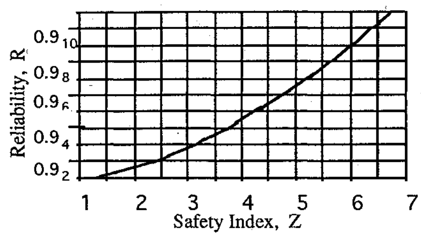

Fig. 2 Reliability vs safety index.

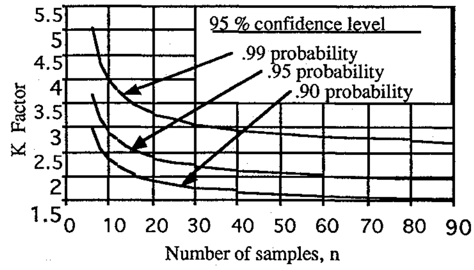

Fig. 3 *K* factor for the normal distribution.

known as the safety index. The relationship between the safety index and the reliability is given by

$$R = P(F_R - F_A > 0) = \phi(Z) \tag{4}$$

where $\phi(Z)$ is the standard cumulative distribution. Figure 2 relates the safety index (3) with the reliability.

While the probability of failure decreases with increasing safety index through the increased difference of the resistive- and applied-stress distribution means, it increases as their combined stress deviations increase. Both conditions govern their tail overlap, as illustrated in Fig. 1 and as expressed by Eq. (3). Unlike the deterministic method, a constant difference of the distribution means may slide up and down the stress axis with no change in tail interference, which verifies that the reliability is independent of the strength of the selected material. The safety index is a total system concept in that all of the probability characteristics of interacting disciplines related to the difference of the means are statistically integrated and uniquely related to an absolute value.

In designing for a specified reliability and its related safety index, Eq. (3) must be characterized with design control and passive variables in common with applied and resistive stress analyses. The deterministic safety factor of Eq. (1) is seen to be bound by the minimum resistive and maximum allowed applied stresses, which are currently defined by lower and upper tolerance limits,[5] respectively. These tolerance limits embody the desired design variables and are cast as

$$A = \frac{F_R}{\mu_R} = 1 - K\eta_R \tag{5}$$

$$B = \frac{F_A}{\mu_A} = 1 + N\eta_A \tag{6}$$

to simplify their presentation and repeated application. Their statistical characterization requires collective understanding.

In selecting a structural material, the statistical mean and standard deviation ($\sigma = \mu\eta$) of the resistive stress are calculated from observed material test data. These are passive design variables. Whereas the same test conducted on the same number of specimens by different experimenters will result in different means and standard deviations, the population must contain results from all of these experiments. To insure, with a certain percentage of confidence, that the given portion is contained in the population, a K factor is applied according to the sample size and proportion. Figure 3 provides random-variable K factors for three probabilities with 95% confidence levels in one-sided normal distributions.

The K factor is a designer-controlled variable in that either an A- or a B-basis material is specified or the number of specimens is selected. Either designer choice determines the K factor. The

A basis requires 99% of materials produced to exceed specified values with 95% confidence, and the B basis allows 90% with the same confidence. The test-derived mean and standard deviation and the associated K factor are substituted in Eq. (5) to characterize the resistive-stress tolerance limit. The maximum allowed applied stress is then ideally equated to the resistive-stress tolerance limit, which may be related to the special case of Eq. (1) by

$$\mathrm{SF} = \frac{F_R}{F_A} = 1 \tag{7}$$

Statistical characteristics of the applied-stress tolerance limit of Eq. (6) are computed from multiaxial limit-load response analysis, which includes the response mean, the standard deviation, and a designer-controlled N factor. The role of these tolerance-limit characteristics in the failure concept is illustrated in Fig. 1 through the division of the stress distribution means into three distinct zones:

$$\mu_R - \mu_A = \lambda_R + \lambda_A + \lambda_0 \tag{8}$$

Zones λ_R and λ_A are the probability ranges of the resistive- and applied-stress tolerance limits of Eqs. (3) and (4), respectively, and zone λ_0 is the error, or margin,

$$\lambda_0 = (\mathrm{SF} - 1)F_A \tag{9}$$

of the nonideal condition of Eq. (7) resulting from design uncertainties. As the test safety factor deviates from unity, the error zone, and therefore the difference of the means, also deviate; all these deviations must be counteracted to satisfy the specified safety index.

Solving for the means and standard deviations from Eqs. (5) and (6), and substituting them and the conditions (7) and (8) into Eq. (3), we obtain the proposed first-order safety index,

$$Z = \frac{\varphi\,\mathrm{SF}\,B - A + (\varphi\,\mathrm{SF} - 1)BA}{\left[(\varphi\,\mathrm{SF})^2\eta_R^2 B^2 + \eta_A^2 A^2\right]^{\frac{1}{2}}} \tag{10}$$

expressed in terms of the six design variables that must satisfy the safety index of the specified reliability.

Once the minimum safety index is specified for a given structural region, the application of design variables to Eq. (10) is essentially the integration of factors developed through interacting disciplines. The safety factor SF is defined by Eq. (7). The material is selected from operational environments and manufacturing considerations. The A- or B-basis structural properties are chosen for the yield limit; together they determine the resistive-stress tolerance-limit factor $1 - K\eta_R$ and its characteristics. The applied-stress tolerance-limit factor $1 + N\eta_A$ is derived and characterized from structural response analysis.

The variables N and K are design control variables, and are currently autonomously derived. Though they may be used to fine-tune Eq. (10) to the specified reliability, a more expedient, versatile, and direct approach is to control the safety index through a compensating coefficient ϕ, combined with the safety factor to yield a modified safety factor ϕ SF, which is obtained from Eq. (10) as

$$\begin{aligned}\varphi\,\mathrm{SF} = \frac{A}{B}\Big((B+1)(A+1) + \big\{[(B+1)(A+1)]^2 \\ - \left[\left(1 - Z^2\eta_R^2\right) + A(A+2)\right]\left[\left(1 - Z^2\eta_A^2\right) + B(B+2)\right]\big\}^{\frac{1}{2}}\Big) \\ \div \left[\left(1 - Z^2\eta_R^2\right) + A(A+2)\right]^{-1}\end{aligned} \tag{11}$$

The design application of the reliability method is thus reduced to determining the modified safety factor, ϕ SF, and applying it to basic stress analyses in place of the conventional safety factor. Since the safety index, safety factor, and material are constant for a structural component, the only design-variable change required by Eq. (11) for calculating the modified safety factor in each region is the response coefficient of variation η_A. The method generates a uniformly reliable structure, and its application requires no new skills and no more exceptional understanding and effort than the prevailing deterministic method. It does require provisions for design uncertainties and an expedient means of safety-index verification for static structures. These matter are discussed in the following sections.

Design Uncertainties

Neglect of design uncertainties is a common cause of premature verification failures. Identifying and estimating recurring structural uncertainty sources, and appropriately implementing them into response models, are essential to a complete design analysis. Incorrect assumptions, faulty software, and other errors that are mendable should not be categorized as uncertainties. Errors that are frequently ignored and that most often depreciate the integrity (lowest quality level for redesign) of a built and tested structure are modeling uncertainties. The four basic types are loads, stress, materials, and manufacturing. The last three modeling uncertainties are exposed by the response of the real test article, and therefore, counteracting margins must be estimated and appropriately implemented as sacrificial factors in design analyses.

Predicted loads applied to test articles must also include the uncertainty margin for load modeling. However, loads and load margins can only be verified downstream in a limited number of all-up surface and flight tests. If subsequent operational or anomalous loads turn out to exceed the design-predicted loads, the inelastic stress region of metallic materials is expected to sustain them in a fail-safe condition to fracture. There is no obvious reason why a similar stress reserve of less than 20 ksi should not be imposed on materials exhibiting linear stress properties to fracture.

Uncertainties that bias the statistical mean must be included as an accumulated uncertainty factor, and random uncertainties may be root-sum-squared (rss) and accumulated:

$$e = e_1 + e_2 + \cdots + \left(e_k^2 + e_{k+1}^2\right)^{0.5} + \cdots + e_n \tag{12}$$

Converging errors from load and stress computational methods and estimated residual stresses from manufacturing techniques are examples of bias in the sample mean of the applied stress. These errors reduce the allowable applied stress by $F_R = (1 + e)F_A$ and must be compensated with a cumulative uncertainty factor, Eq. (12), acting on the applied stress mean.

Modeling uncertainties that are statistically characterized variables and are mutually exclusive may be defined as a multivariable function by combining their dispersions through the following error propagation laws.[6] When two or more independent variables are added, their standard deviations are rss by the summation function rule,

$$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} \qquad \text{for} \quad z = x + y \tag{13}$$

When independent variables are multiplied and/or divided, their coefficients of variation are rss according to the power function rule,

$$\eta_z = \sqrt{n^2\eta_x^2 + m^2\eta_y^2} \qquad \text{for} \quad z = x^n y^m \tag{14}$$

Structures designed with mean properties and having uncertainty dispersions about the design means are in this class of error propagation. Manufacturing and material stock tolerances are examples of the summation function rule. The elastic modulus is determined by a number of variables having measured dispersions and should be combined by the power function rule. Poisson's ratio is another commonly applied mean property whose wide dispersions are particularly critical in bending and fracture mechanics analyses.

This type of uncertainty is usually associated with stress and manufacturing dispersions and may be combined with the applied stress coefficient of variation derived from response analysis. Its neglect reduces the combined coefficient of variation,

$$\eta_{Ae} = 2\eta_A - \left(\eta_A^2 + \eta_e^2\right)^{0.5} \tag{15}$$

A similar expression may apply to postponed resistive-stress dispersions. The applied-stress tolerance limit factor of Eq. (6) is then adjusted to include propagation uncertainties of Eq. (15) and is expressed by

$$B_e = 1 + N\eta_{Ae} \tag{16}$$

Design Application

Combining the propagation errors with the applied-stress dispersions of Eq. (15), compensating the error zone λ_0 for the cumulative error as discussed, and retaining all other variables in Eq. (10), the constant safety index is seen to be satisfied by

$$Z = \frac{(\varphi_D \, \mathrm{SF})B_e - A + AB_e[(\varphi_D \, \mathrm{SF}) - e - 1]}{\left[\eta_R^2(\varphi_D \, \mathrm{SF})^2 B_e^2 + \eta_{Ae}^2 A^2\right]^{\frac{1}{2}}} \tag{17}$$

with a revised "design-to" compensating coefficient ϕ_D. The modified safety factor is found as before from Eq. (17):

$$\begin{aligned} \phi_D \, \mathrm{SF} = {} & \frac{A}{B_e}\Big[[B_e(1+e)+1](A+1) \\ & + \Big(\{[B_e(1+e)+1](1+A)\}^2 - \Big[\left(1 - Z^2\eta_R^2\right) + A(A+2)\Big] \\ & \times \Big\{\left(1 - Z^2\eta_{Ae}^2\right) + B_e(1+e)[2 + (1+e)B_e]\Big\}\Big)^{\frac{1}{2}}\Big] \\ & \div \Big[\left(1 - Z^2\eta_R^2\right) + A(A+2)\Big]^{-1} \end{aligned} \tag{18}$$

Verifiable Safety Index

In verifying a static structural safety index, it should be recognized that the true test-article response is not the ideally assumed resistive- and applied-stress relationship of Eq. (7). The ratio of the observed test response to the ideal is expected to be greater or less than unity from one article to another. Since this ratio identically defines the deterministic test safety factor of Eq. (1) based on the NASA yield-stress criterion, and because data are available from most structural static tests, the deterministic test-derived safety factor

$$\mathrm{SF}_T = \frac{F_R}{F_A} \qquad \text{for} \quad \mathrm{SF}_T > 0 \tag{19}$$

should be an opportune test variable for concurrently verifying the safety index of static structures. Since estimated uncertainty variables are expected to affect the true response of the test structure, the formulation of the net verified safety index Z_T must include all uncertainty and design-variable values and the compensating coefficient ϕ as applied in Eqs. (17) and (18), except the safety factor SF_T derived from deterministic tests. Using Eq. (17), the safety index is seen to be

$$Z_T = \frac{(\varphi_D \, \mathrm{SF}_T)B_e - A + AB_e[(\varphi_D \, \mathrm{SF}_T) - e - 1]}{\left[\eta_R^2(\varphi_D \, \mathrm{SF}_T)^2 B_e^2 + \eta_{Ae}^2 A^2\right]^{\frac{1}{2}}} \tag{20}$$

Safety-Index-Criteria

The most compelling demand for a structural-reliability method is for supporting risk analyses and associated management. Though selection criteria are still based on sparse and sketchy concepts, one approach may be to define the cost of risk and balance it with the initial and recurring cost of not exceeding that risk. The cost of risk may be defined as the product of the probability of failure, $p = 1 - R$, and the cost of failure, which includes costs of life and property loss, costs of operational and experiment delays, and inventory cost. Initial costs used in the risk analysis include that of the increased structural sizing, based on a reliability design method, to provide reliability. Recurring costs include increased propellant and increased cost due to payload performance loss caused by the increased structural sizing and propellant weights to accommodate the risk side of the balance.

A more immediate demand for a simple and user-friendly reliability method is to supplement current safety audits based on safety-factor margins alone. Designer confidence in the transition to the proposed reliability method may be enhanced by formulating the safety index of Eq. (10) in terms of familiar deterministic design variables, for the sake of comparison and to phase in a spectrum of values representative of successful deterministic experiences. A first-cut safety index was bounded with a small sample of A-basis materials, 3σ probability forcing-function dispersions, and design variables associated with a current aerostructure. The resulting minimum safety index exceeded a value of 4 at the operational stress limit. But more importantly, the analysis indicated that the reliability was most sensitive to the test-derived safety factor, which confirmed its role as the verification variable. The safety index was an order of magnitude less sensitive to other variables. This limited analysis suggested development of a generalized safety-index criterion based on desensitizing and bounding representative design variables.

The proposed safety-index method is based on a maximum allowed linear operating stress. If the index is extended to fracture of metallic materials, the condition (5) must be equated to the ratio of ultimate to yield stress to avoid operating in the inelastic stress range and causing permanent deformation. This extended condition would theoretically increase the distribution tail lengths while decreasing their interference. But, in fact, the data content in those long thin tails should be suspect, and the predicted increased reliability is ill defined. There are pragmatic and compelling physical reasons for designing, verifying, and operating within the linear stress region of the material, and for testing to fracture.

Conclusions

The conventional pass-fail deterministic method does not support risk analyses of semistatic structural designs. Since the method ignores probability contributions independently specified by material and load tolerance limits, it is seen to provide neither comparative nor comprehensive stress audits. Though structures may be designed to a common safety factor, variations in the load tolerance limit from one article or region to another produce nonuniform reliability valuations. The safety factor provides the distance one may dare to stand from a cliff's edge. The probabilities that the structure's weakest material will support the maximum standing load must be integrated with the safety factor to characterize the total system reliability.

A verifiable first-order reliability concept is proposed to overcome these deficiencies for metallic and composite structures. The concept combines the benefits of first-order reliability and deterministic methods, and reduces their application to a modified safety factor to be used instead of the conventional deterministic safety factor. It defines and appropriately implements design uncertainties. It verifies the structural response through a test factor, utilizing existing analytical and computational design techniques to facilitate load-stress-sizing iteration cycles. It requires no more special skills, labor, and organization than prevailing deterministic methods derived from conventional deterministic test methods. The ensuing closed-form reliability solutions may be directly applied to existing analytical and computational design techniques.

The study has provided a detailed prescription for a first-order reliability application and verification methods, using mutual deterministic design variables and current structural computational codes. Results may be confidently phased into the dominantly practiced deterministic stress audits as a substitute, for independent appraisal, or to complement them.

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